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TITLE: TWO-LENS, ANAMORPHIC, BREWSTER-ANGLE, FOURIER-TRANSFORM RELAY

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Abstract

A two-lens system provides a simple and versatile means to relay a laser beam. The pair of lenses can provide true volume imaging, reproducing both amplitude and phase of the input beam. By using cylindrical lenses it is possible to change the aspect ratio of the beam. By adjusting the cylindrical curvatures, it is possible to minimize reflections by tilting the lenses at the Brewster angle.

Introduction

Such a relay pair was developed in the course of the design of optics for a photochemical process. The laser produces a square, multi-mode beam. The reaction zone has the shape of a rectangular solid, with a height/width aspect ratio of approximately three. Because the beam has a high intensity, reflections, damge to optical coatings, and thermal distortions in the optics were important issues. All of these potential problems can be minimized by the use of only two thin, tilted, uncoated lenses.

Single-lens beam relays

Under certain conditions, a single lens is adequate for relaying a beam. Two examples are shown in Figure 1. If the laser beam is Gaussian (having a single lateral mode), then the relay shown in Figure 1a is all that is needed. The second waist has exactly the same properties as the first waist; the beam will propagate beyond the second waist with the same intensity and phase distribution that occurs beyond the first waist. By using different spacings the waist can be magnified or reduced.

If the beam is not Gaussian with a single lateral mode, then the intensity distribution can still be imaged exactly with a single lens, as shown in Figure 1b. The second plane is the image of the first. In this case the phase distribution is not reproduced and the beam will not propagate beyond the second plane with the same intensity as the beam beyond the first plane.
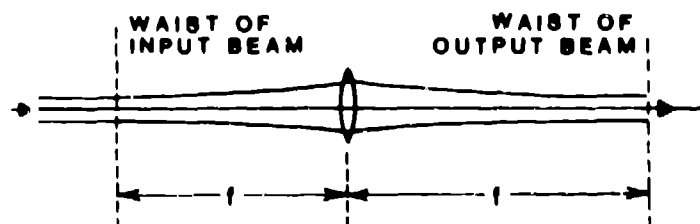
a) WAIST-TO-WAIST PROPAGATION
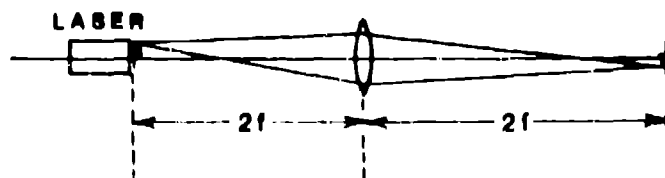
b) LASER IMAGED ONTO A SECOND PLANE

Figure 1. Single-lens relays.

This can be understood geometrically by considering the output of the laser, at the first plane, as a combination of collimated beamlets at varying angles. Each beamlet will then have a focal point between the lens and the second plane and will be diverging, not collimated, at the second plane.

If we want to relay the laser volume into another volume, either for a stage of amplification or for a photochemical reaction, this approach may not be adequate.

Two-lens beam relay

A pair of lenses separated by the sum of their focal lengths, as shown in Figure 2, will image a volume exactly. This has been termed an exact Fourier transform relay[1] because the first lens creates an exact Fourier transform of the beam at the entrance pupil; and that is again transformed by the second lens. As shown in Figure 3, collimated light emerges collimated, and the entrance pupil is imaged to an exit pupil. Furthermore, plane A is imaged to A' and plane B to B', demonstrating the volume-imaging characteristic of such a system. The relaying will be exact for either Gaussian beams or any arbitrary intensity distribution.[2]
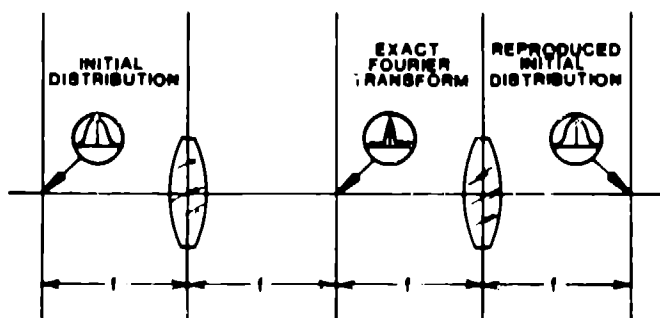

Figure 2. Exact Fourier transform relay.
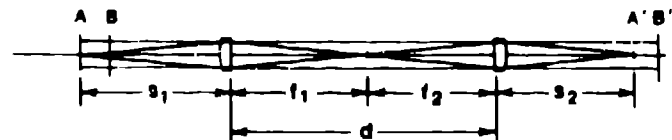
Figure 3. Image properties of two-lens relay.

The figure shows a symmetric case, having the entrance and exit pupil distances, s_1 and s_2 equal. An extreme non-symmetric case occurs when the entrance pupil is placed at the first lens. Then for 1:1 relaying, the second lens falls at an image plane. The elements then act as relay lens and field lens in a conventional periscope configuration. For a periscope this is a nice configuration, minimizing the needed tube diameter. For a laser relay, the symmetric case is desired because it keeps the focus away from all elements.

To obtain the volume-imaging property, it is not necessary to have a 1:1 relay. For a given magnification M we must have

$$f_2/f_1 = M \qquad\qquad (1)$$

For the afocal condition,

$$f_1 + f_2 = d \qquad\qquad (2)$$

Solving the lens equation to maintain an image of the entrance pupil, while keeping the center section afocal, given

$$s_1 M + s_2/M = d \qquad \text{or} \qquad s_2 = M (d-Ms_1) \qquad\qquad (3)$$

The first volume is imaged into the second with a lateral magnification of M and a longitudinal magnification of M^2. There are four variables (s_1, s_2, f_1, and f_2), and only three equations. Therefore a range of entrance pupil spacings (s_1) can meet the conditions for any desired value of d.

For unit magnification, $f_1 = f_2 = f$, the solution given by equation (2b) reduces simply to $s_1 + s_2 = 2f$.

Anamorphic relay

The two lenses can have cylindrical components, giving different focal lengths in the x-z plane and the y-z plane (taking the propagation to be in the z-direction). Then the equations given above can be met in both planes simultaneously, with different magnifications M_x and M_y. Using x and y subscripts for the two planes gives, for the magnifications:

$$f_{2x}/f_{1x} = M_x \tag{4}$$

$$f_{2y}/f_{1y} = M_y \tag{5}$$

Since there is a fixed distance between the two lenses,

$$f_{1x} + f_{2x} = f_{1y} + f_{2y} = d \tag{6}$$

From these three equations we get

$$f_{1x} = d/(M_x+1) \tag{7a}$$

$$f_{1y} = d/(M_y+1) \tag{7b}$$

Rewriting equation (3) for the two planes gives:

$$s_2 = M_x(d-M_x s_1) \tag{8a}$$

$$s_2 = M_y(d-M_y s_1) \tag{8b}$$

If $M_x = M_y$, then, as in equation (3), the two equations can be satisfied for a given d with a range of values of s_1. However, if $M_x \neq M_y$, these can only be satisfied if

$$s_1 \approx d/(M_x + M_y) \tag{9}$$

and

$$s_2/s_1 = M_x M_y \tag{10}$$

In general there will be a constraint on the overall length. There will then be one combination of s_1, s_2, and d that meets all requirements.

An example having $M_y = \sqrt{2}$ and $M_x = 1/\sqrt{2}$ is shown in Figure 4. For this ray trace, cylindrical surfaces have been used, placing the x-curvatures on the front surface of each lens and the y-curvatures on the rear surfaces. The upper figure shows the afocal operation on collimated rays. The separated foci in the x-z and y-z planes are visible between the two lenses. The lower figure shows the imaging of one pupil onto another.

Using tilted lenses

Tilting a spherical lens will give a larger decrease in the effective focal length in one plane than in the other. By using equal tilts in orthogonal planes, the effect of cylindrical lenses shown in Figure 4 can be obtained with spherical surfaces. Figure 5 shows an anamorphic relay system using lenses tilted at 30 degrees.

In this figure the y-z and x-z planes are shown separately. In each part of the figure, ray traces show the afocal and pupil-imaging properties. The anamorphic ratio is about 1.8:1.
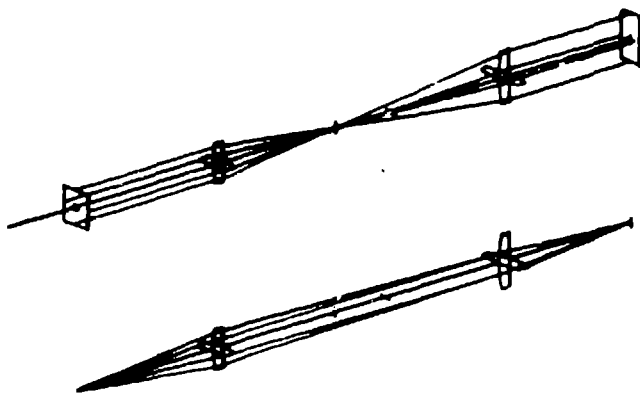

y-z PLANE

x-z PLANE

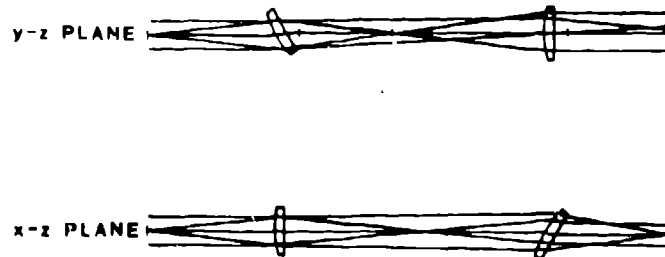Figure 4. Anamorphic cylindrical relay, Ratio 2:1.

Figure 5. Anamorphic tilted relay, 30 degrees.

Brewster-angle relays

If the beam is polarized, it is useful to place components at the Brewster angle. Since both lenses will then be tilted in the same plane, the tilt will not introduce anamorphic distortion. For this case, we again use cylindrical components, choosing the proper cylindrical curvatures for the tilted components. Figure 6 illustrates a system of this type.

Aberrations

It needs to be emphasized that these designs were developed for the purpose of relaying small, high-power laser beams over large distances, a case for which diffraction effects are important. For such cases, the f/numbers of the lenses are of the order of 100 or higher. Furthermore, since the beams are generated from a multimode laser, they are not in general of extremely high quality. In such cases, the aberrations of the cylindrical and tilted components are insignificant.

For the illustrations shown in this paper we have used very short systems in order to illustrate the ray paths. The various systems are shown scaled to an overall length of 200 mm and an entrance pupil diameter of 10 mm. At this scale, the cylindrical anamorph shown in Figure 4 has nearly 10 waves of aberration. Even so, the ray aberrations are a fraction of a millimeter, indicating that in some conditions it might be acceptable for relaying a 10-mm diameter beam. Scaled by a factor of 10 to a more realistic 2-m overall length, the aberration drops to less than 1/10 wave for the same 10-mm entrance pupil.

The Brewster-angle system was originally designed for an overall relay length of 10 m, an input beam size of 10 mm, and an anamorphic magnification ratio, M_y/M_x of 3. The aberrations were on the order of 1/100 wave. With an overall length of 4 m, probably the shortest useful length, the aberration still remains less than 1/2 wave.

y-z PLANE

x-z PLANE

Figure 6. Brewster-angle anamorphic relay, ratio 3:1.

Discussion

The two-lens relay gives a reproduction of the phase and intensity of the beam at the laser, limited only by aperture clipping and aberrations. As a result the beam will propagate in the region of interest with the minimum possible divergence. The two-lens system can produce beam shaping by giving different magnifications in the two planes. Since the lenses can be placed at Brewster's angle, no coatings will be needed for a polarized beam. Reflections will be nearly eliminated, and any that occur will be directed out of the beam. Thus this system provides a nearly perfect solution to this relaying problem.

References

1. E. Sklar, "Fourier-transform Ring Laser," J. Opt. Soc. Am. **A1**, 537 (1984).
2. W. D. Gunter, Jr. and A. DeYoung, "New Properties of Afocal Lens Pairs with Gaussian Beams," Appl. Opt. **25**, 1742 (1986).